УДК 624.074.436

ISMAEL TAHA FARHAN
IMOMNAZAROV T. S.

The theoretical specification of stiffened conoidal shell roof with pointload

Исмаил Таха Фархан

аспирант департамента строительства Инженерной академии Российского университета дружбы народов, Москва

e-mail: ismael.civilengineer@gmail.com

Имомназаров Тимур Соибназарович

аспирант департамента строительства Инженерной академии Российского университета дружбы народов, Москва

e-mail: timur-imomnazarov@mail.ru

A review of literature reveals that shell structures are applicable in many fields of mechanical engineering have been studied by many researchers. Conoidal shells which are very popular as civil engineering roofing units have not received due attention. Conoidal shell configurations are aesthetically appealing, structurally stiff, and they may be used for covering large column free spaces. The greatest advantage of conoids from the construction point is that they are easy to cast as the surfaces are ruled. These shells provide uniform lighting to the covered area, and are more suitable when greater rise is needed at one end. Naturally, these forms received importance from the engineers and research on conoidal shells dates back to seventh decade of the twentieth century. Static characteristics of a stiffened conoidal shell have been studied in this paper. For static analysis of stiffened conoidal shell roofs different shell actions such as deflection, force and moment resultants have been computed.

Keywords: conoidal shell, mathematical formulation, shell element, compatibility, finite element method, formulating static problem.

ИСМАИЛ ТАХА ФАРХАН, ИМОМНАЗАРОВ Т. С.
ТЕОРЕТИЧЕСКАЯ СПЕЦИФИКАЦИЯ ЖЕСТКОЙ КОНОИДАЛЬНОЙ КРЫШКИ СО СОСРЕДОТОЧЕННОЙ НАГРУЗКОЙ

Обзор литературы показывает, что оболочки применяются во многих областях машиностроения. Коноидальные оболочки, которые очень популярны в архитектуре, не получили должного внимания в научных исследованиях. Коноидальные оболочки являются выразительными, структурно жесткими и могут применяться для покрытия больших пространств без колонн. Как линейчатые оболочки, их без особых трудностей можно монтировать. Они обеспечивают равномерное освещение крытой площади и более подходят, когда на одном конце требуется большее увеличение. Естественно, коноидальные оболочки пользуются большим интересом у инженеров и исследователей с семидесятых годов двадцатого столетия. В данной работе изучены статические характеристики упрочненной коноидальной оболочки. Для статического анализа упрочненных крыш с коноидальной оболочкой были раскрыты различные действия оболочки, такие как деформации, силы и моменты.

Ключевые слова: коноидальная оболочка, математическая формулировка, отсек оболочек, совместимость, метод конечных элементов, формулировка статической задачи.

Overview of literature

The reason of a shell form having high strength with small thickness can be attributed to the coupling of in-plane thrusts and bending of its surface under external superimposed load. A conoid is generated by a variable straight line moving parallel to a plane-known as the director plane-with one of its ends on a plane curve and the other on a straight line. Depending on the arrangement of stiffeners with respect to the shell midsurface the disposition can be classified as concentric or eccentric. The stiffeners disposition is concentric if the centroid of the stiffener cross section coincides with the shell midsurface and is eccentric if the stiffener centroid remains above or below the shell mid-surface. From structural aspect the concrete conoidal shell roofs are advantageous as these are singly ruled and hence are easy to cast. The doubly curved conoidal shells are architecturally appealing and therefore preferred to cover large column-free spaces in many industrial applications like airport lobbies, automobile manufacturing and repairing yards and so on. Conoidal shells are efficient in admitting natural light and ventilation. These shells need to be stiffened to achieve greater strength-to-weight ratio.

The use of stiffened structural elements started in the nineteenth century perhaps, first being Howland and Beed (1940) tested cylindrical shells stiffened

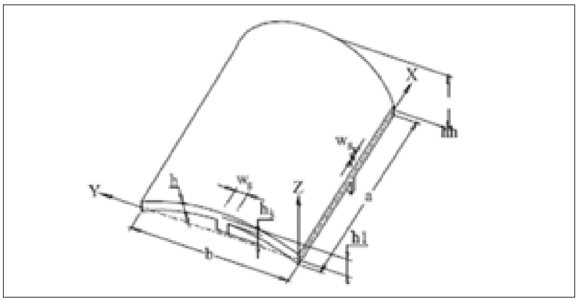


Figure 1: Orientation of the Shell in the Global Cartesian coordinate system.

$$Z = 4\left[hl + (hh - hl)\frac{x}{a}\right]\left[\frac{y}{b} - \frac{y^2}{b^2}\right]$$

where

a, b — Length and width of shell in plan

X, Y, Z — global coordinate axes

h, h_s — Shell thickness and stiffener thickness respectively

hh, hl — higher and lower heights of conoid respectively

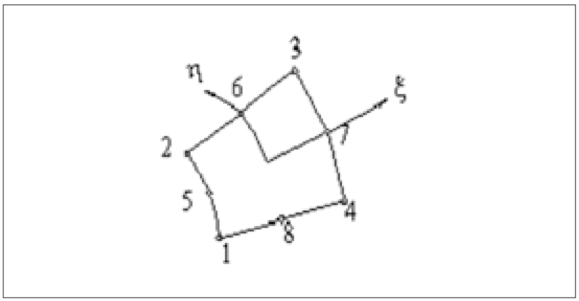


Figure 2: Eight nodded curved Quadratic element

orthogonally by equally spaced stringers and rings subjected to internal pressure. Soare (1959) [1], Ramaswamy (1961), Rao (1961), Bhise et al (1962), developed the membrane theory to find the stress resultants of various types of conoidal shells with different loading and boundary conditions [2].

They also attempted gradual simplifications of the theoretical formulations. Hadid (1964) used a variation method to study the bending characteristics of elastic conoids and analyzed both simply supported and clamped conoids using an integral equation technique. Choi (1984) modified the quadratic isoparametric element with the help of four extra non-conforming displacement modes added only to transverse displacements for analyzing thin shells. The extra modes were finally condensed out. Choi used this element for analyzing conoidal shell problems solved earlier by Hadid (1964). Ghosh and Bandyopadhyay (1989) reported a detailed static analysis of isotropic conoidal shells giving results of deflections, forces and moments. They used an eight noded isoparametric doubly curved shallow shell finite element in their analysis. Most of the latest literatures (A.N. Nayak & J.N. Bandyopadhyay, 2006), S. Kumari & D. Chakravorty (2010) have dealt with static characteristics of clamped stiffened conoidal shell for different number and orientation of stiffeners. Suman Kumari & D. Chakravorty (2011) have considered eight noded isoparametric shell element for analyzing the bending behaviour of delaminated composites conoidal shells under uniformly distributed load with various practical boundary conditions. In this paper a static analysis of stiffened conoidal shell subjected to concentrated load has been presented by using finite element method.

Mathematical formulation

For this mathematical formulation eight noded quadratic isoparametric shell element & a three noded curved beam element is used for solution by finite element method. In the present analysis the surface of stiffened shell is discretized into a number of finite elements, which is further considered as a combination of shell and beam elements. Formulation of the general curved shell element, having all the three radii of curvature Rx, Ry and Rxy using eight noded isoparametric quadratic elements, is derived. The stiffeners are modeled as three noded curved beam elements having the same set of displacement functions as that of the shell excluding the one across the stiffener longitudinal axis. Appropriate combinations of the element matrices of the shell and the beam elements results in the overall element matrices of the stiffened shell element are presented.

Shell element

A doubly curved thin shallow shell of uniform thickness made of homogenous isotropic linearly

Elastic material is considered. The orientation of the shell in the global Cartesian coordinate system is show in Figure 1. The shell surface is discretized by curved quadratic elements, which are rectangles in plan. These rectangles are modeled as eight noded doubly curved isoparametric elements having four corner and four midside nodes Figure 2. The natural coordinate system of ξ and η of the isoparametric element, which is connected to the Cartesian coordinate system through the Jacobian matrix [3].

Shape functions

For an isoparametric element the coordinates and displacements at any point within the element are represented by the coordinates and displacements of the nodes of the element and the shape functions. These are derived from an interpolation polynomial [4]. The interpolation polynomial is a function of ξ and η and has the following from.

$$u(\xi,\eta) = A_0 + A_1\xi + A_2\eta + A_3\xi^2 + A_4\eta + A_5\eta^2 + \\ +A_6\eta + A_7\eta^2 \quad (1)$$

where

$A_{0...}A_7$ — Constants of displacement polynomial

ξ, η — Local natural coordinates of an element

The shape functions derived from the interpolation polynomial are as [5], [7].

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1) \quad i = 1,2,3,4$$
$$N_i = \frac{1}{2}(1+\xi\xi_i)(1+\eta^2) \quad i = 5,7$$
$$N_i = \frac{1}{2}(1+\eta\eta_i)(1+\xi^2) \quad i = 6,8 \quad (2)$$

where

N_i denotes the shapes function at i^{th} node having natural coordinates $\xi\xi_i$ and $\eta\eta_i$.

The coordinates of any point (x, y) within the element are obtained as

$$x = \sum N_i x_i \text{ and } y = \sum N_i y_i \quad i = 1,\dots\dots,8 \quad (3)$$

Where x_i and y_i are the coordinates of the i^{th} node.

Generalized displacement fields and nodal degrees of freedom

A shell surface is modeled by three-dimensional solid elements. The thickness is considerably smaller than the other dimensions and the nodes along the thickness direction need additional degrees of freedom and hence are not preferred [6]. When a two-dimensional element is obtained by condensing the thickness direction nodes, the displacements of adjacent thickness direction nodes must be ensured to be equal to avoid numerical difficulties. Thus five degrees of freedom including three translations (u, v, w) and two rotations (α, β) are attached to each node. The final element has midsurface nodes only and a line in the thickness direction remains straight but not necessarily normal to the mid surface after deformations. The generalized displacements at any point within the element can be interpolated from the nodal values as

$$\{\delta\} = \begin{Bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} N_i & & & & \\ & N_i & & & \\ & & N_i & & \\ & & & N_i & \\ & & & & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} \tag{4}$$

Equation (4) can be written in a compact form

$$\{\delta\} = [N]\{d_e\} \tag{5}$$

where

u, v, w displacements along x, y and z axes respectively

α, β Rotations vectorially along y and x axes respectively

$\{d_e\}$ Vector containing element nodal displacements and rotations

Strain-displacement equations

According to the modified Sanders' first approximation theory for thin shells (Sanders, 1959) [2], the strain-displacement relationships are established as

$$\{\varepsilon\} = [B]\{d_e\} \tag{6}$$

where

$\{\varepsilon\}$ = strain matrix

$[B]$ =strain-displacement matrix and

$\{d_e\}$ =nodal displacement vector

Force-strain relationships

The force and moment resultants are obtained from the stresses as

$$\{F_e\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_x z \\ \sigma_y z \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{7}$$

where σ_x and σ_y are the normal stresses along X and Y directions, respectively and τ_{xy}, τ_{xz} and τ_{yz} are shear stresses in XY, XZ and YZ planes, respectively [8], [9]. The thickness of the shell is denoted by h. The force-strain relation can be further derived from the stress-strain relations and finally can be presented as

$$\{F\} = [D]\{\varepsilon\} \tag{8}$$

where

$[D]$ Elasticity matrix of shell element

Stiffener element

An isoparametric curved three-node beam element is chosen with two end nodes and one middle node to model the stiffeners [10]. The beam elements are oriented in natural coordinate system along ξ or η parallel to the global X or Y axes respectively, as shown in Figure 3 & Figure 4.

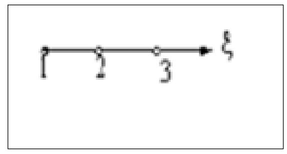


Figure 3: X-stiffener

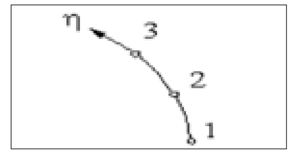


Figure 4: Y-Stiffener

In the beam elements [11], each node has four degrees of freedom, u^{sx}, v^{sx}, α^{sx} and β^{sx} for x directional stiffeners and u^{sy}, v^{sy}, α^{sy} and β^{sy} for y-directional stiffeners. The shape function, strain-displacement equation, and force strain relation of stiffener element has been derived as in case of shell element considering four degrees of freedom.

Compatibility

In order to maintain compatibility between the shell and beam elements, the stiffener nodal degrees of freedom have to be transformed to shell degrees of freedom considering the eccentricity and curvature of the stiffener. Considering the effect of eccentricity and curvature the axial displacement of any point in the midsurface of the x-stiffener u^{sx} can be expressed in terms of the axial displacement u of a point in the midsurface of the shell as [12]

$$u^{sx} = u\left[1 + \frac{c}{R_x}\right] + e\alpha \tag{9}$$

Or in compact form

$$\{\delta^{sx}\} = [T_{ce}^{sx}]\{\delta\}$$

Formulating static problem

The expression for static equilibrium can be stated as

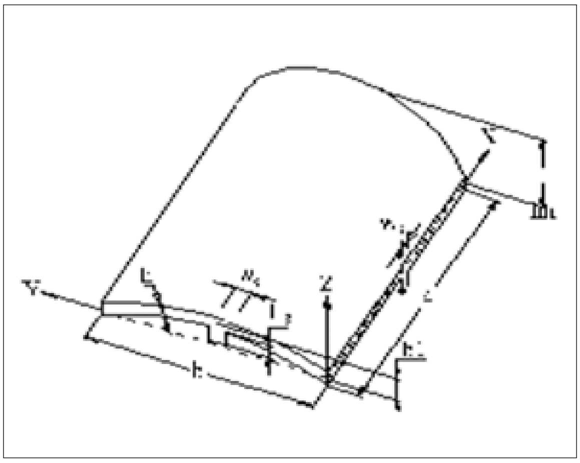
$[K]\{d\} = \{Q\}$

The above equation is solved by the Gauss elimination technique and from the global nodal displacement vector $\{d\}$ thus obtained, the element displacement vector $\{d_e\}$ is known [6].

Effect of stiffener eccentricity

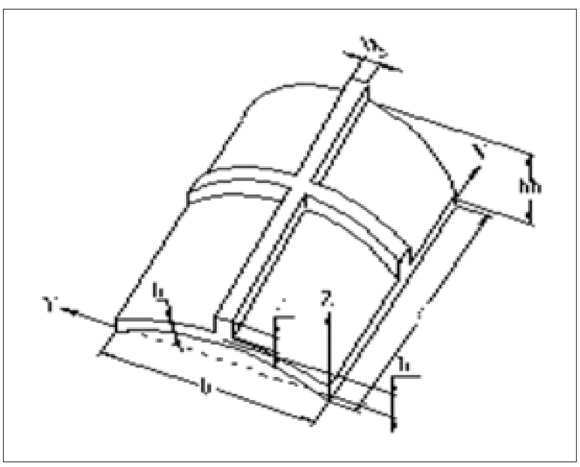
The eccentricity is designated as negative or positive depending on whether the stiffener lies below or above the shell surface respectively. The eccentricity is zero when the mid surfaces of the shell and the stiffener coincide. It is observed that shells with positive or negative eccentricity exhibit almost equal values of all shell actions and hence a designer has the flexibility of providing the stiffeners above or below the shell surface being governed by functional requirements only [7].

Eccentrically stiffened shells are better than concentrically stiffened including the transverse deflection and the moments which normally govern the shell depth. However, for compressive in-plane forces and tensile, the shell with $\frac{e}{h} = 0$ exhibits a better performance. The major in-plane compressive force which also has a role in determining the shell depth shows the lowest value for $\frac{e}{h} = 0$. It is also extremely interesting to note that the points of occurrences of the critical shell actions are identical whether eccentricity is positive, negative or zero in almost all cases. The few exceptions to this general trend are noted for tensile and hogging moments for concentrically stiffened shells only.



$$z = 4\left[hl + (hh - hl)\frac{x}{a}\right]\left[\frac{y}{b} - \frac{y^2}{b^2}\right]$$

Figure 5: Conoidal shell with eccentric stiffeners (negative eccentricity)



$$z = 4\left[hl + (hh - hl)\frac{x}{a}\right]\left[\frac{y}{b} - \frac{y^2}{b^2}\right]$$

Figure 6: Conoidal shell with eccentric stiffeners (Positive eccentricity)

Conclusions

The mathematical formulation as per Sanders' shallow shell theory applied here for conoidal shell can be confidently recommended to analyze static characteristics of stiffened ruled surfaces. The curved stiffeners along y-direction are more efficient in reducing the deflections of conoids compared to the straight ones along x-direction. For bare and x-stiffened shells the clamped edge gives much lesser deflections than the simply supported edge. When stiffeners are provided along y-direction or both directions the deflections of clamped and simply supported shells come relatively closer. Eccentrically stiffened shells are found to be better choices than the concentrically stiffened ones and whether the eccentricity of stiffeners is positive or negative matters very little. The relative performance matrices will provide a rational approach to select a particular shell option from many possibilities, a decision often a practicing engineer has to make. These papers are presented to serve as important design aids to practical engineers.

References

1 J. L. Sanders, 1959, an improved first-approximation theory for thin shells, NASA Technical Report TR-24.
2 B. Budiansky and J. L. Sanders, 1963, on the «best» first order linear shell theory, Prog. Appl. Mech. (Prager Anniversary Vol.), MacMillan, NY, 129–140.
3 Analysis of Geometrically Nonlinear Structures Second Edition by Robert Levy Technion — Israel Institute of Technology, Haifa, Israel and William R. Spillers New Jersey Institute of Technology, Newark, New Jersey, U. S. A.
4 Theory of Shell Structures C. R. CALLADINE structural mechanics, university of Cambridge and fellow of peterhouse.
5 Differential Geometry of Curves and Surfaces Victor Andreevich Toponogov with the editorial assistance of Vladimir Y. Rovenski.
6 Gil-oulbé Mathieu. The stress-strain analysis of epitrochoidal shells. — PhD thesis — Moscow — 1997.
7 Eduard Ventsel &Theodor Krauthammer, Thin Plates and Shells Theory, Analysis, and Applications, by Marcel Dekker, 2001.
8 Jawad, Maan H. Design of plate and shell structures, by The American Society of Mechanical Engineers, 2004.
9 Szilard, Rudolph, Theories and applications of plate analysis: classical, numerical and engineering methods, by John Wiley & Sons, 2004.
10 Advanced mechanics of materials: Plates, http://madhuvable.org/wp content/uploads/2015/11/amom-chapter-5-slides.
11 ANSEL C. UGURAL, Stresses In Beams, Plates, And Shells, Third Edition, CRC Series in Computational Mechanics and Applied Analysis, 2011, http://www.crcpress.com.
12 J. N. Reddy, Theory and Analysis of Elastic Plates and Shells Second Edition, 2007 by Taylor & Francis Group, http://www.taylorandfrancis.com.